Diffusion Models for Visual Computing

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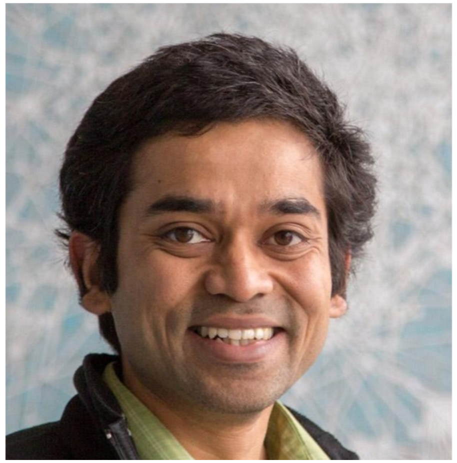
Part 1: Introduction to Diffusion Models

UCL Adobe TEL AVIV UNIVERSITY

https://geometry.cs.ucl.ac.uk/courses/diffusion4VC_eg24/

People



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Why do we need this Tutorial?

What are diffusion model?

What are the design choices?

Controls and adaptation in the context of Visual Computing

Learn together

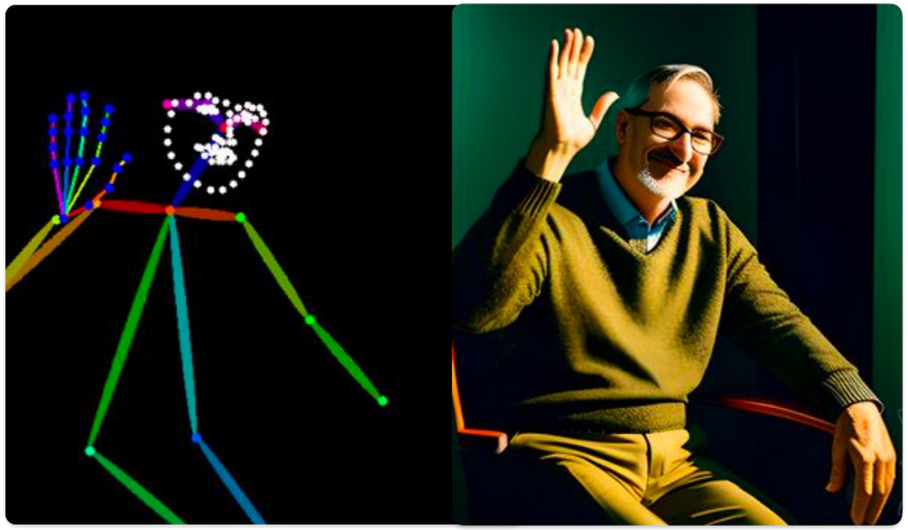
Related Materials

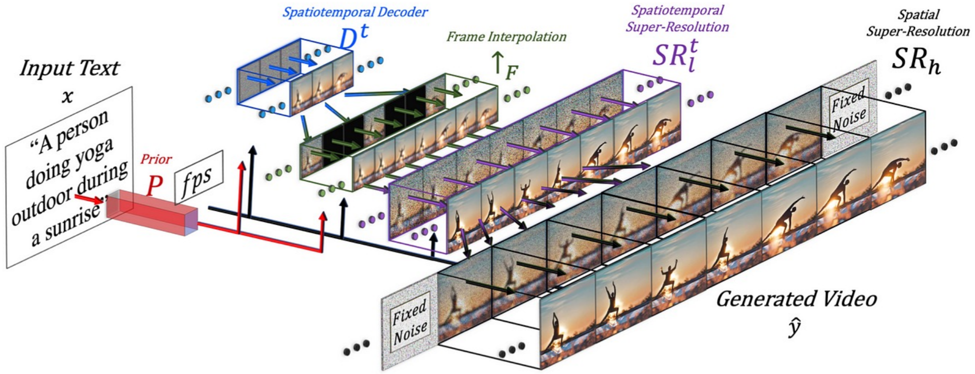
- Survey papers
- Past tutorials/courses
- Blogs and recorded videos

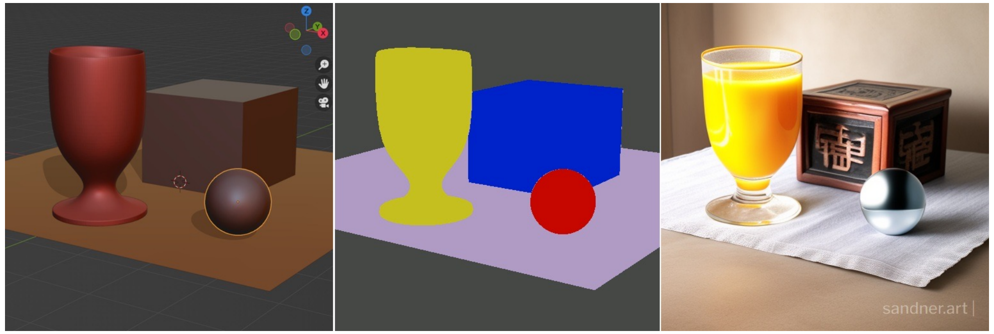
Presentation Schedule

Introduction to Diffusion Models
Guidance and Conditioning Sampling
Attention
Break
Personalization and Editing
Beyond Single (RGB) Image Generation
Diffusion Models for 3D Generation

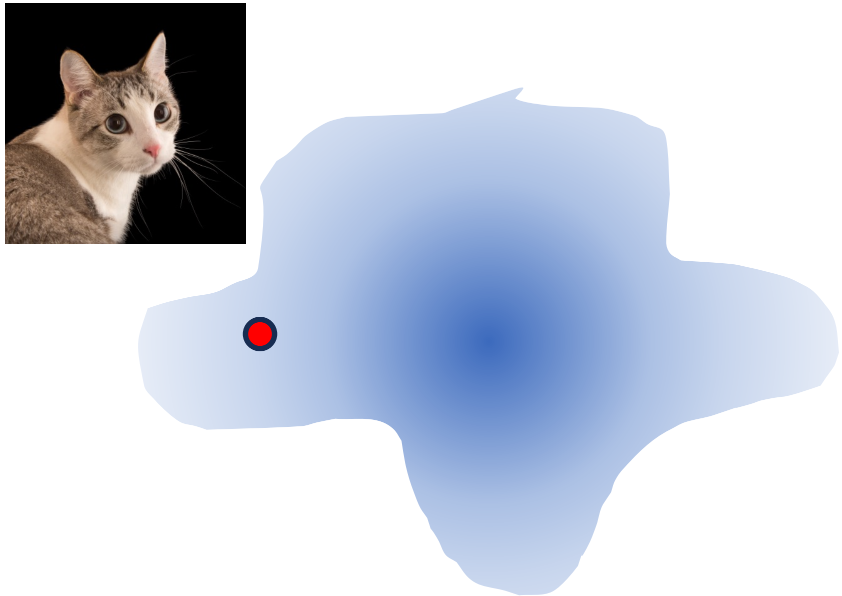
Applications



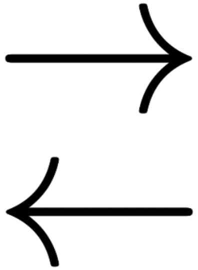




What is a Diffusion Process?



unknown map



(unknown) data distribution

known distribution

Sampling ⇔ (Unconditional generation)

Mapping between Distributions

$$\mathbf{x}_0 \longrightarrow \mathbf{x}_T$$

$$\mathcal{N}(\mathbf{0}, \mathbb{I})$$

data distribution

known distribution

Gaussian (Normal) Distribution

- Uniquely defined by Mean and Variance

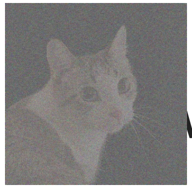
$$\mu, \boldsymbol{\Sigma}$$

- Reparameterization 'trick'

$$\mathcal{N}(\mu, \boldsymbol{\Sigma}) \qquad x \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$$

- Many results on combining Gaussian distributions

Mapping in Many Steps



ward mapping

$$\mathbf{x}_0 \rightarrow \quad \mathbf{x}_{t-1} \rightarrow \mathbf{x}_t \quad \rightarrow \quad \rightarrow \mathbf{x}_T$$

$$\mathcal{N}(\mathbf{0}, \mathbb{I})$$

$$q(x_t|x_{t-1}) = \mathcal{N}(x_t; \sqrt{\alpha_t}x_{t-1}, (1-\alpha_t)\mathbb{I})$$

Mapping in Many Steps

$\longrightarrow$

forward mapping

$$\mathbf{x}_0 \rightarrow \quad \mathbf{x}_{t-1} \rightarrow \mathbf{x}_t \quad \rightarrow \quad \rightarrow \mathbf{x}_T$$

$$\mathcal{N}(\mathbf{0}, \mathbb{I})$$

$$q(x_t|x_{t-1}) = \mathcal{N}(x_t; \sqrt{\alpha_t} x_{t-1}, (1-\alpha_t)\mathbb{I})$$

Mapping in Many Steps

$\longrightarrow$

forward mapping

$$\mathbf{x}_0 \rightarrow \quad \mathbf{x}_{t-1} \rightarrow \mathbf{x}_t \quad \rightarrow \quad \rightarrow \mathbf{x}_T$$

$$\mathcal{N}(\mathbf{0}, \mathbb{I})$$

$$q(x_t|x_{t-1}) = \mathcal{N}(x_t; \sqrt{\alpha_t} x_{t-1}, (1-\alpha_t)\mathbb{I})$$

$$q(x_t|x_0) = \mathcal{N}(x_t; \sqrt{\overline{\alpha}_t} x_0, (1-\overline{\alpha}_t)\mathbb{I}) \qquad \overline{\alpha}_t = \Pi_{i=1}^{t} \alpha_i$$

Mapping in Many Steps

$\longrightarrow$

forward mapping

$$\mathbf{x}_0 \rightarrow \quad \mathbf{x}_{t-1} \rightarrow \mathbf{x}_t \quad \rightarrow \quad \rightarrow \mathbf{x}_T$$

$$\mathcal{N}(\mathbf{0}, \mathbb{I})$$

$$q(x_t|x_0) = \mathcal{N}(x_t; \sqrt{\bar{\alpha}_t}x_0, (1-\bar{\alpha}_t)\mathbb{I})$$

$$x_t = \hat{\epsilon}(x_0) = \sqrt{\bar{\alpha}_t}x_0 + \sqrt{(1-\bar{\alpha}_t)}\epsilon_t \qquad \bar{\alpha}_t = \Pi_{i=1}^{t}\alpha_i$$

Generative Modeling: Sampling

$\longleftarrow$

reverse mapping

$$\mathbf{x}_0 \rightarrow \quad \mathbf{x}_{t-1} \rightarrow \mathbf{x}_t \quad \rightarrow \quad \rightarrow \mathbf{x}_T$$

$$\mathcal{N}(\mathbf{0}, \mathbb{I})$$

$$x_t = \sqrt{\overline{\alpha}_t} x_0 + \sqrt{(1 - \overline{\alpha}_t)} \epsilon_t$$

$$q(x_{t-1} | x_t)$$

$$\approx$$

$$p(x_{t-1} | x_t) = \mathcal{N}(x_{t-1}; \sqrt{\overline{\alpha}_{t-1}} D_\theta(x_t, t), (1 - \overline{\alpha}_{t-1}) \mathbb{I})$$

Loss Functions

$$\mathcal{L}_{simple}(\theta) = \mathbb{E}_{t,x_0,\epsilon}[C_t \parallel \epsilon_\theta(x_t, t) - \epsilon \parallel^2]$$

$$\mathcal{L}(\theta) = \mathbb{E}_{t,\epsilon,x_0}\left[C_t \parallel D_\theta(\hat{\epsilon}_t(x_0), t) - x_0 \parallel^2\right]$$

$$p(x_{t-1}|x_t) = \mathcal{N}(x_{t-1}; \sqrt{\bar{\alpha}_{t-1}} D_\theta(x_t, t), (1 - \bar{\alpha}_{t-1})\mathbb{I})$$

Algorithm (How to Train?)

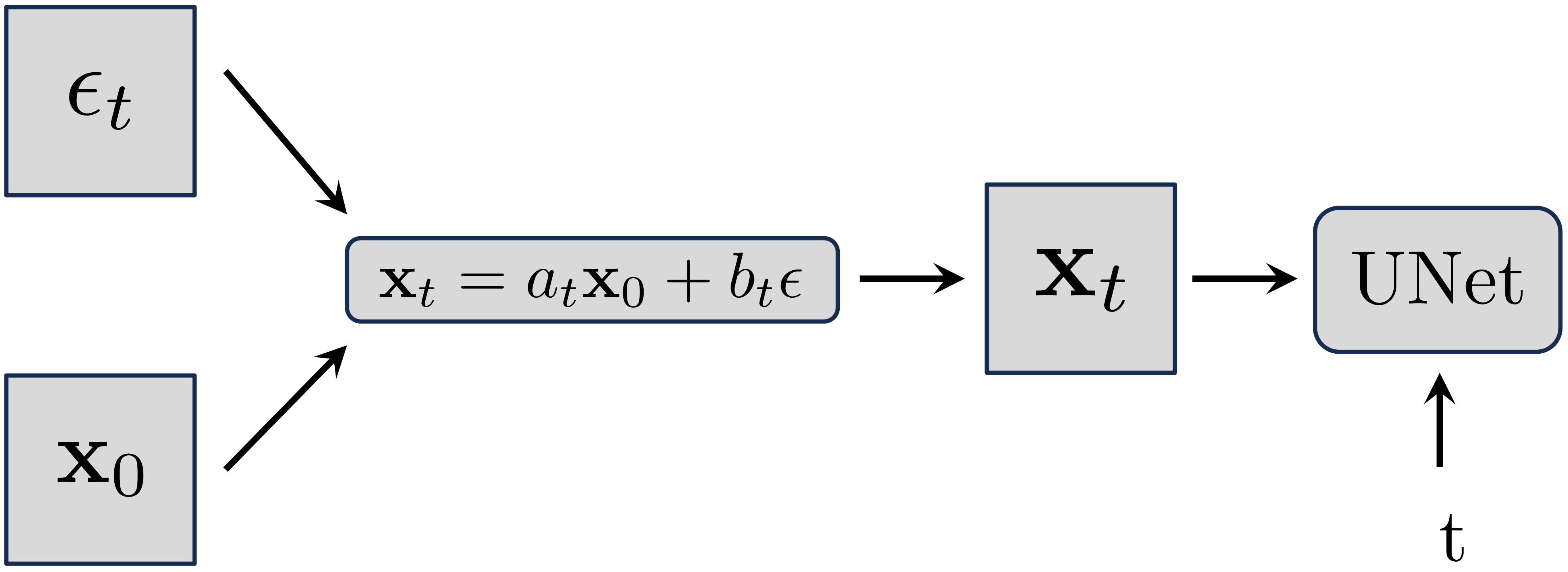
Algorithm 1 Training

1: **repeat**
2: $\mathbf{x}_0 \sim q(\mathbf{x}_0)$
3: $t \sim \mathrm{Uniform}(\{1, \ldots, T\})$
4: $\boldsymbol{\epsilon} \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$
5: Take gradient descent step on

$$\nabla_\theta \left\| \boldsymbol{\epsilon} - \boldsymbol{\epsilon}_\theta(\sqrt{\bar{\alpha}_t}\mathbf{x}_0 + \sqrt{1 - \bar{\alpha}_t}\boldsymbol{\epsilon}, t) \right\|^2$$

6: **until** converged

Training Loss



Three Interpretations

- Predict Noise ϵ_t

- Predict clean image $\mathbf{x}_0$

- Score-based optimization $\nabla_{\mathbf{x}_t} \log p(\mathbf{x}_0) = -\frac{\epsilon}{\sqrt{1-\bar{\alpha_t}}}$

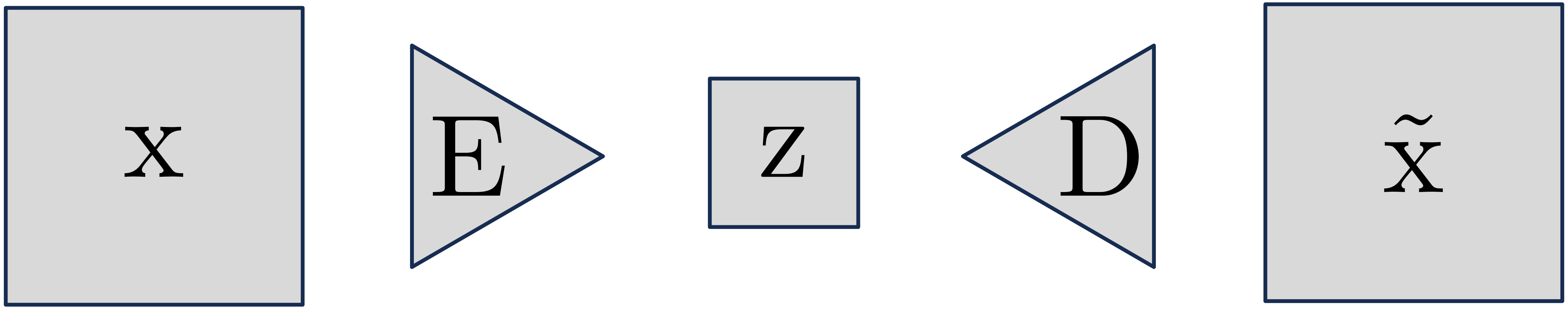
they are equivalent!!

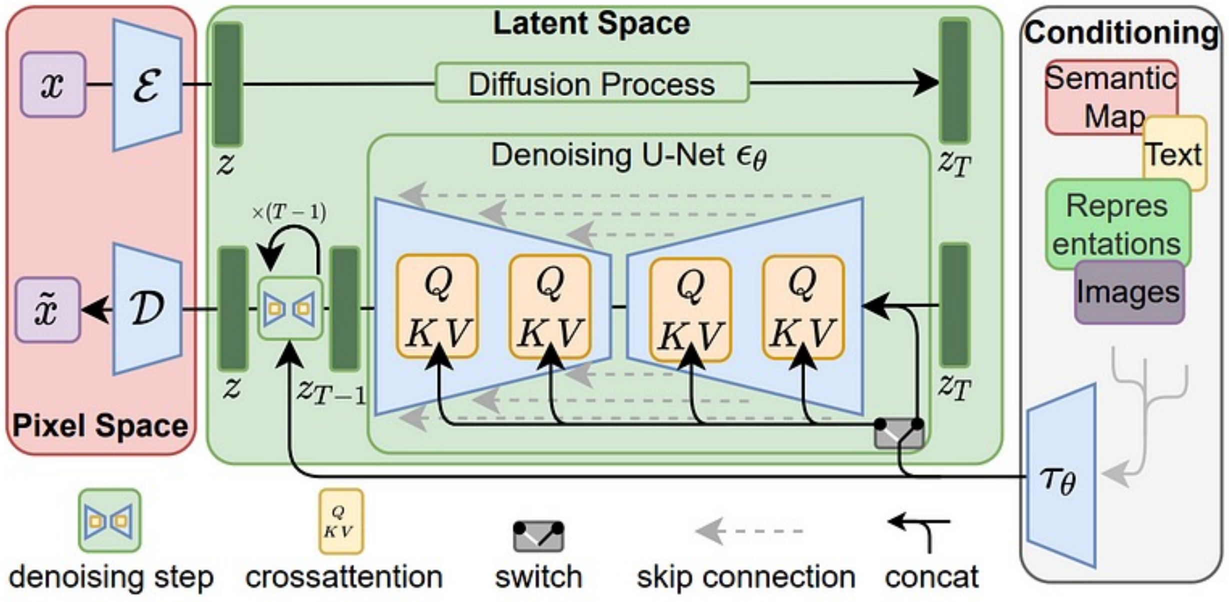
Algorithm (How to Sample?)

Algorithm 2 Sampling

1: $\mathbf{x}_T \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$
2: **for** $t = T, \ldots, 1$ **do**
3: $\quad \mathbf{z} \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$ if $t > 1$, else $\mathbf{z} = \mathbf{0}$
4: $\quad \mathbf{x}_{t-1} = \frac{1}{\sqrt{\alpha_t}} \left(\mathbf{x}_t - \frac{1-\alpha_t}{\sqrt{1-\bar{\alpha}_t}} \boldsymbol{\epsilon}_\theta(\mathbf{x}_t, t) \right) + \sigma_t \mathbf{z}$
5: **end for**
6: **return** $\mathbf{x}_0$

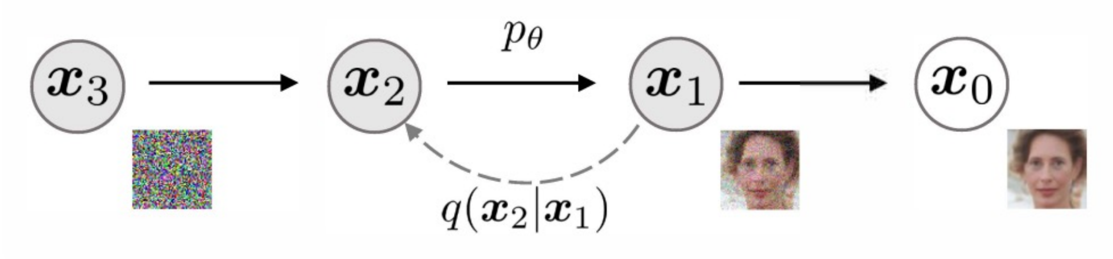
Latent Diffusion Model



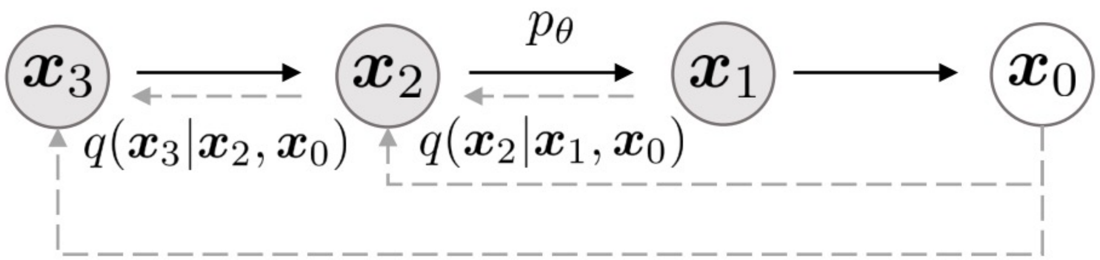


DDPM vs DDIM

- DDPM: Markovian process



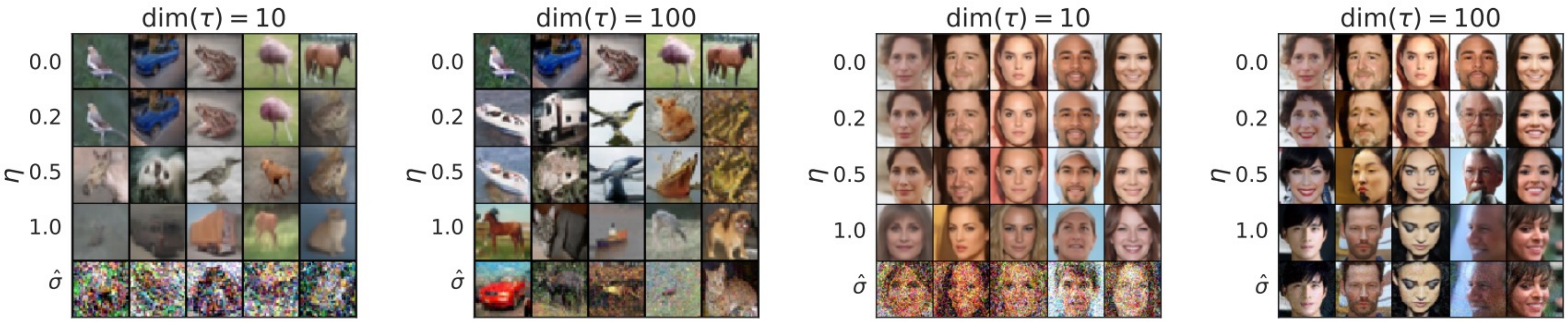
- DDIM: Non-Markovian process but 10-50x faster!!
  – Trained w/ pretrained DDPM diffusion



$$\boldsymbol{x}_{t-1} = \sqrt{\alpha_{t-1}} \underbrace{\left(\frac{\boldsymbol{x}_t - \sqrt{1-\alpha_t}\epsilon_\theta^{(t)}(\boldsymbol{x}_t)}{\sqrt{\alpha_t}} \right)}_{\text{`` predicted } \boldsymbol{x}_0\text{''}} + \underbrace{\sqrt{1-\alpha_{t-1}-\sigma_t^2} \cdot \epsilon_\theta^{(t)}(\boldsymbol{x}_t)}_{\text{``direction pointing to } \boldsymbol{x}_t\text{''}} + \underbrace{\sigma_t \epsilon_t}_{\text{random noise}}$$

DDPM vs DDIM

		CIFAR10 (32 × 32)					CelebA (64 × 64)			
S		10	20	50	100	1000	10	20	50	100	1000
η	0.0	**13.36**	**6.84**	**4.67**	**4.16**	4.04	**17.33**	**13.73**	**9.17**	**6.53**	3.51
	0.2	14.04	7.11	4.77	4.25	4.09	17.66	14.11	9.51	6.79	3.64
	0.5	16.66	8.35	5.25	4.46	4.29	19.86	16.06	11.01	8.09	4.28
	1.0	41.07	18.36	8.01	5.78	4.73	33.12	26.03	18.48	13.93	5.98
$\hat{\sigma}$		367.43	133.37	32.72	9.99	**3.17**	299.71	183.83	71.71	45.20	**3.26**



Summary so far

Algorithm 1 Training

repeat
  $\mathbf{x}_0 \sim q(\mathbf{x}_0)$
  $t \sim \mathrm{Uniform}(\{1, \ldots, T\})$
  $\boldsymbol{\epsilon} \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$
  Take gradient descent step on
  $$\nabla_\theta \left\| \boldsymbol{\epsilon} - \boldsymbol{\epsilon}_\theta(\sqrt{\bar{\alpha}_t}\mathbf{x}_0 + \sqrt{1-\bar{\alpha}_t}\boldsymbol{\epsilon}, t) \right\|^2$$
until converged

Algorithm 2 Sampling

$\mathbf{x}_T \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$
for $t = T, \ldots, 1$ **do**
  $\mathbf{z} \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$ if $t > 1$, else $\mathbf{z} = \mathbf{0}$
  $\mathbf{x}_{t-1} = \frac{1}{\sqrt{\alpha_t}} \left(\mathbf{x}_t - \frac{1-\alpha_t}{\sqrt{1-\bar{\alpha}_t}} \boldsymbol{\epsilon}_\theta(\mathbf{x}_t, t) \right) + \sigma_t \mathbf{z}$
end for
return $\mathbf{x}_0$

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