EG25 Supplemental

October 2024

1 Analytical Test Surfaces

Here we provide the formulas for the four test surfaces used in the paper.

If U, V are spherical polar coordinates and $P(U,V)$ represents the Cartesian coordinates of the corresponding point on the sphere, i.e. $P(U,V) = (\sin U \cos V, \sin U \sin V, \cos U)$ then the corresponding point on the analytic surface is given by the following formulas (the NumPy formulas are also given, for ease of implementation):

$$\mathrm{r}_{mushroom}(U,V) = 0.301\left(1 + 0.05\left(2U^3 \sin^2(6U) + 0.6\left(U(\pi - U)\right)^2 \sin^2\left(6V + 6U^2\right)\right)\right) P(U,V)$$

```
0.301 * (P * np.transpose(( np.transpose(np.ones_like(P)) + 0.05 *
(2*((U)**3)*np.sin(6*U)**2 + 0.6*((U*(np.pi-U))**2)*np.sin(6*V + 6*(U)**2)**2)
) ))
```

$$\mathrm{r}_{bobble}(U,V) = 0.679\left(1 + 0.1\left(0.4\left(U(\pi - U)\right)^2 \cos^2(12U) + 0.4\left(U(\pi - U)\right)^2 \sin^2(12V + 6U)\right)\right) P(U,V)$$

```
0.679 * (P * np.transpose(( np.transpose(np.ones_like(P))
+ 0.1 * (0.4*((U*(np.pi-U))**2)*np.cos(12*U)**2 + 0.4*((U*(np.pi-U))**2)
*np.sin(12*V + 6*U)**2) )))
```

$$\mathrm{r}_{star}(U,V) = 0.772\left(1 + 0.4(1 - \cos(U))(1 + \cos(U))\sin(6U)\sin(6V)\right) P(U,V)$$

```
P * np.transpose(( np.transpose(np.ones_like(P))
+ 0.4*(1-np.cos(U))*(1+np.cos(U))*np.sin(6*U) * np.sin(6*V) ))
```

$$\mathrm{r}_{flower} = 0.366\left(1 + 0.3\left(0.4\left(U(\pi - U)\right)^2 \cos^2(18U) + 0.6\left(U(\pi - U)\right)^2 \sin^2(6V + 18U)\right)\right) P(U,V)$$

```
P * np.transpose(( np.transpose(np.ones_like(P))
+ 0.3 * (0.4*((U*(np.pi-U))**2)*np.cos(18*U)**2 + 0.6*((U*(np.pi-U))**2)*np.sin(6*V
+ 18*U)**2) ))
```

We provide the MatLab output (not fully simplified) for various differential quantities in the files:

- `mushroom.txt`
- `flower.txt`
- `bobble.txt`
- `star.txt`

The files contain the expressions for the components of the First Fundamental Form — `E`, `F`, `G`, the Second Fundamental Form — `L`, `M`, `N`, the normal — `n1`, `n2`, `n3`, the minimum curvature direction — `dir11`, `dir12`, `dir13` and the maximum curvature direction — `dir21`, `dir22`, `dir23`.

2 Figure 11 Scalar Field

The equation for the 'frequency-varying sine wave' scalar field, used as a test case in Figure 11, is:

$$f(\mathbf{x}) = s\sin(m(\omega, \mathbf{x}, \mathbf{n})) \tag{1}$$

where s is an amplitude parameter, ω is a frequency parameter and $\mathbf{n}$ is a direction parameter, and $m(\omega, \mathbf{x}, \mathbf{n}) := 3\omega((\mathbf{x} \cdot \mathbf{n})^2 + 0.5)$.

The Euclidean gradient is

$$\nabla f(\mathbf{x}) = 6sf(\mathbf{x} \cdot \mathbf{n})\cos(m(\omega, \mathbf{x}, \mathbf{n}))\mathbf{n} \tag{2}$$

and the Hessian is

$$\mathbf{H}(\mathbf{x}) = 6sf\mathbf{n}\mathbf{n}^T \left(\cos(m(\omega, \mathbf{x}, \mathbf{n})) - 6f(\mathbf{x} \cdot \mathbf{n})^2 \sin(m(\omega, \mathbf{x}, \mathbf{n}))\right). \tag{3}$$

We use Equation 17 to compute the result of the LB operator applied to this function. The quantities that Equation 17 involves are:

- ∇f, $\mathbf{H}(f)$ - computed analytically as above
- The (Euclidean) Laplacian Δf - this is the trace of $\mathbf{H}(f)$.
- The mean curvature and the normals to the SNS, computed via the formulas in section 3.

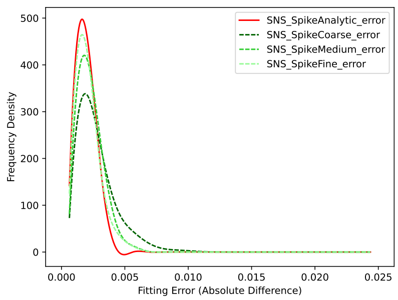
When the scalar field is neural (as described at the start of section 5) the only difference is that the Euclidean gradient and Hessian are computed by autograd, instead of analytically.

3 NFGP Note

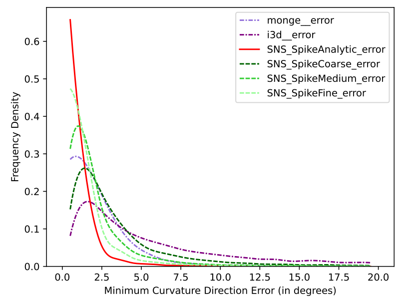
We have had some problems training NFGP with the author's implementation. We found that in several cases the reconstruction quality started to degrade rather than improve after a small number of epochs. We used the default settings. We could not run it for the full 300 epochs (estimated time 45 hours) due to compute constraints and it would possibly converge to a good solution given the full time. The results that we show in Figure 7 are for the best checkpoint within the first 10000 iterations (10 epochs).

Reconstructions show that NFGP struggled to fit to thin structures present in the 'flower' model and fine surface detail present on the 'mushroom' shape. The 'flower' reconstruction contained many holes and we believe that this can be because their sampling method it not appropriate for a shape with very thin parts.

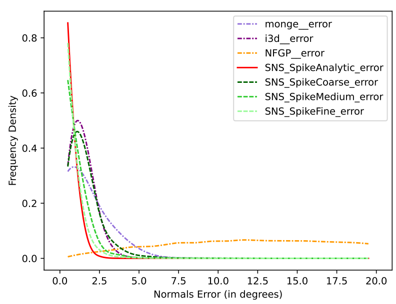
4 Additional Error Plots



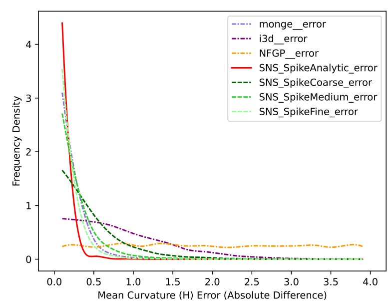
(a) Geometry Error



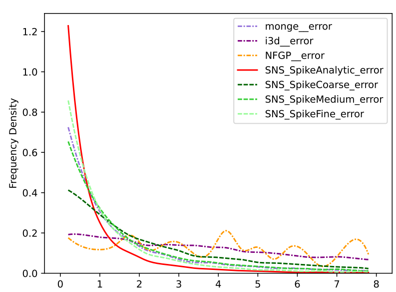
(b) Min Curv. Dir Error



(c) Normals Error

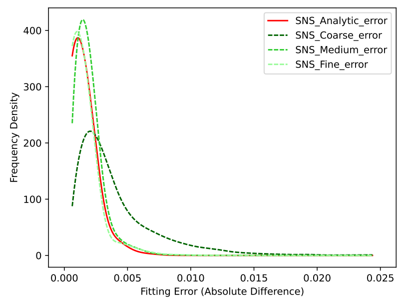


(d) H Error

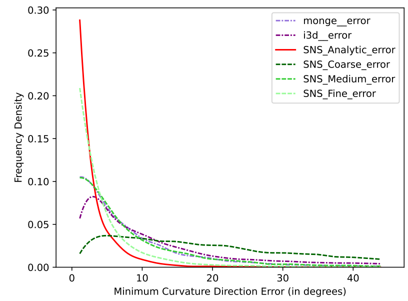


(e) K Error

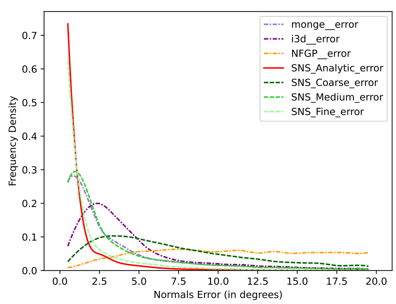
Figure 1: ‘Star’ Error Plots



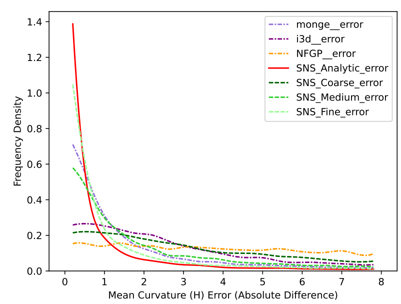
(a) Geometry Error



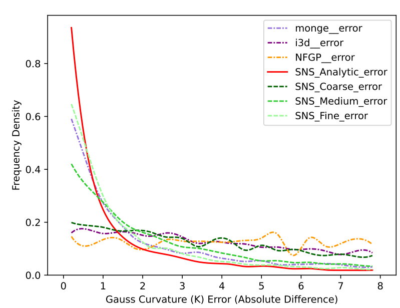
(b) Min Curv. Dir Error



(c) Normals Error

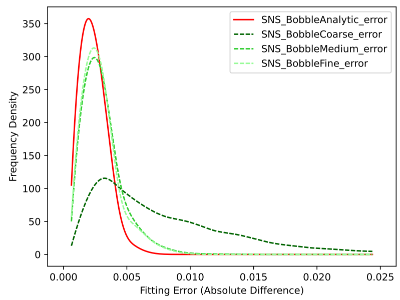


(d) H Error

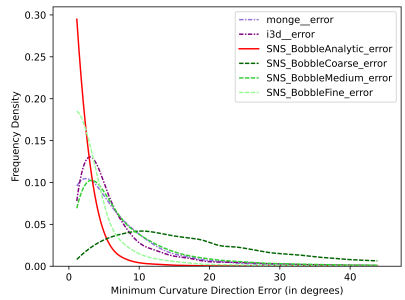


(e) K Error

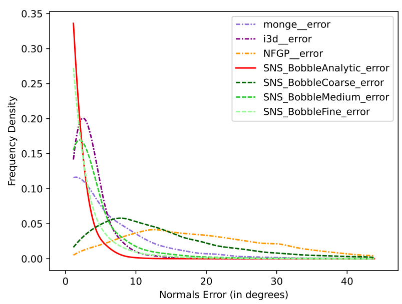
Figure 2: 'Mushroom' Error Plots



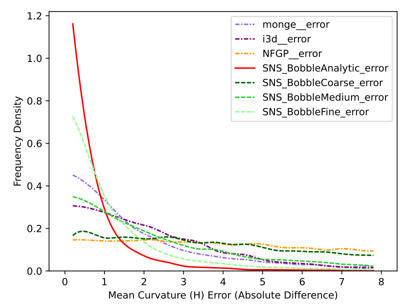
(a) Geometry Error



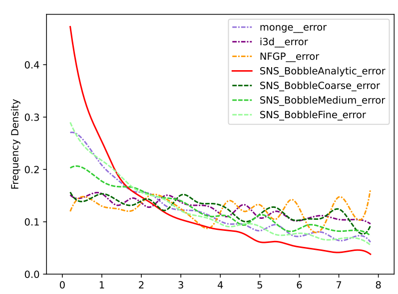
(b) Min Curv. Dir Error



(c) Normals Error

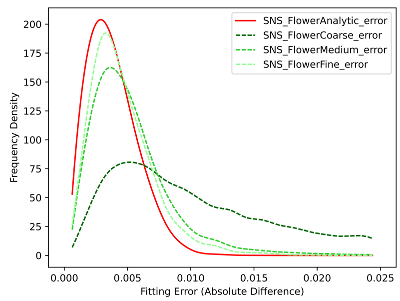


(d) H Error

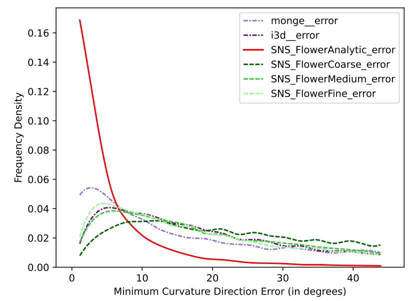


(e) K Error

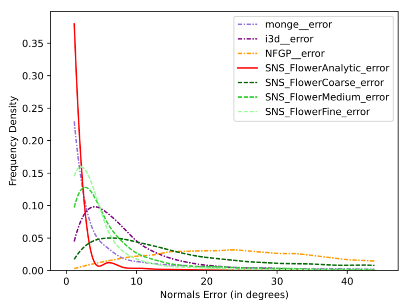
Figure 3: 'Bobble' Shape Plots



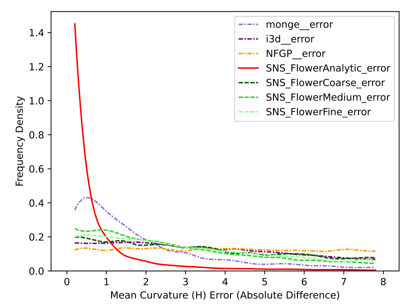
(a) Geometry Error



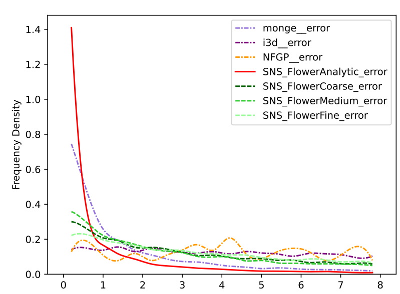
(b) Min Curv. Dir Error



(c) Normals Error



(d) H Error



(e) K Error

Figure 4: ‘Flower’ Shape Plots